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Dilaton tadpoles and mass in warped models

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Abstract

We review the brane world sum rules of Gibbons at al for compact five dimensional warped models with identical four-geometries and bulk dynamics involving scalar fields with generic potential. We show that the absence of dilaton tadpoles in the action functional of the theory is linked to one of these sum rules. Moreover, we calculate the dilaton mass term and derive the condition that is necessary for stabilizing the system.

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1 Introduction

Recently Gibbons, Kallosh and Linde [1] derived an infinite set of sum rules for five dimensional models with a compact periodic extra dimension and identical four geometries. These constraints were an immediate consequence of the equations of motion and served as consistency checks of several recent constructions. An interesting result was that the Goldberger-Wise (GW) mechanism [2] of stabilizing the Randall-Sundrum (RS) model [3] has to include the backreaction on the metric in order to agree with a specific constraint, something done in the DeWolfe-Freedman-Gubser-Karch mechanism [4].

A particular sum rule that attracted the attention of [1] was the $\oint dy W(T_\mu^\mu - 2T_5^5) = 0$, where $W(y)^2$ stands for the warp factor. This constraint was firstly derived by [5] as a condition of vanishing of the four dimensional cosmological constant. The interesting point was that this combination of the energy-momentum tensor components appeared in a condition for the absence of dilaton² tadpoles in the action functional of the theory in the paper by Kanti, Kogan, Olive and Pospelov [6]. The condition of [6] reads $\oint dy \sqrt{G^{(5)}}(T_\mu^\mu - 2T_5^5) = 0$. It was however pointed out by [1] that the two constraints were not identical because the $(T_\mu^\mu - 2T_5^5)$ combination was integrated with different powers of the warp factor since $\sqrt{G^{(5)}} = W^4 \sqrt{g}$. A closer inspection reveals that these two constraints are actually identical given the assumptions made in [6]. In more detail, the condition in [6] was derived for matter dominated branes where the effect of the warp factor is negligible. Then, for $W \approx 1$ the two constraints coincide.

In this paper we will iterate the calculation of [6], including the full effect of the warp factor. In that case, the condition of [6] for the absence of dilaton tadpoles is modified and the new condition coincides with an other sum rule of [1]. Furthermore, having the quadratic action functional for the dilaton, it is straightforward to read off its mass. We find a generic formula relating the dilaton mass with the sum of the tensions of the two branes and the curvature of the four-geometries. Demanding that this mass is not tachyonic we can derive the necessary condition for stabilizing the overall size of the system. This is in accordance with the result found in [1] that the GW stabilization mechanism of the RS model has to include the backreaction on the metric.

²Here we use the term “dilaton” to denote the modulus corresponding to the fluctuation of the overall size of the system. We use instead the term “radions” for the moduli associated with the position of freely moving branes along the extra dimension (not on orbifold fixed points).

2 Review of sum rules

At first it would be instructive to review the sum rules presented in [1]. We will concentrate as in [1] in the case where the background metric of the five dimensional spacetime can be written in the form:

$$ds^2 = W(y)^2 g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2 \quad (1)$$

with $g_{\mu\nu}(x)$ a general background four dimensional metric and $W(y)$ a generic warp factor. We should stress here that this is not the most general choice of metric in five dimensions as we have explicitly assumed that the all four dimensional sections have the same geometry.

We can now consider an arbitrary number of minimally coupled scalar bulk fields $\Phi^I(x, y)$ with internal metric \mathcal{G}_{IJ} and arbitrary bulk potential $V(\Phi)$ (which includes bulk cosmological constant), coupled to an again arbitrary number of branes with brane potential $\lambda_i(\Phi)$ (which again includes the brane tensions). The action describing the above system is the following:

$$S = \int d^4x dy \sqrt{-G} \left(2M^3 R - \frac{1}{2} \mathcal{G}_{IJ} \partial_M \Phi^I \partial^M \Phi^J - V(\Phi) - \sum_i \lambda_i(\Phi) \delta(y - y_i) \frac{\sqrt{-\hat{G}^{(i)}}}{\sqrt{-G}} \right) \quad (2)$$

where $\hat{G}_{\mu\nu}^{(i)}$ is the induced metric on the brane and M the fundamental 5D scale. The Einstein equations arising from the above metric can be written in the form:

$$4M^3 R_\mu^\mu = -\frac{1}{3} T_\mu^\mu - \frac{4}{3} T_5^5 \quad (3)$$

$$4M^3 R_5^5 = -\frac{1}{3} T_\mu^\mu + \frac{2}{3} T_5^5 \quad (4)$$

where the energy-momentum tensor components are:

$$T_\mu^\mu = -\partial_\mu \Phi \cdot \partial^\mu \Phi - 2\Phi' \cdot \Phi' - 4V(\Phi) - 4 \sum_i \lambda_i(\Phi) \delta(y - y_i) \quad (5)$$

$$T_5^5 = -\frac{1}{2} \partial_\mu \Phi \cdot \partial^\mu \Phi + \frac{1}{2} \Phi' \cdot \Phi' - V(\Phi) \quad (6)$$

with the indices in the above formulas raised and lowered by $G_{\mu\nu} = W(y)^2 g_{\mu\nu}(x)$ and where dot product denotes construction with the internal metric \mathcal{G}_{IJ} . Since we are interested on

a background configuration, the $\partial_\mu \Phi \cdot \partial^\mu \Phi$ terms can be dropped. The Ricci tensor is easily calculated to be:

$$R_\mu^\mu = W^{-2} R_g - 12W'^2 W^{-2} - 4W'' W^{-1} \quad (7)$$

$$R_5^5 = -4W'' W^{-1} \quad (8)$$

If we now consider the function $(W^a)''$ with a an arbitrary real number, its integral around the compact extra dimension is zero. Using (3), (4), (7), (8) we arrive at an infinite number of constraints [1]:

$$\oint dy W^a (T_\mu^\mu + (2a - 4)T_5^5) = 4M^3(1 - a)R_g \oint dy W^{a-2} \quad (9)$$

As it is obvious, these constraints are a natural consequence of the equations of motion. It is straightforward to see for example that they are satisfied in the RS model [3] as well as the bigravity/multigravity models [9]. We will single out three constraints which we will be important for the subsequent discussion, namely the ones for $a = 0, 1, 2$:

$$\oint dy (T_\mu^\mu - 4T_5^5) = 4M^3 R_g \oint dy W^{-2} \quad (10)$$

$$\oint dy W (T_\mu^\mu - 2T_5^5) = 0 \quad (11)$$

$$\oint dy W^2 T_\mu^\mu = -4M^3 R_g \oint dy \quad (12)$$

3 Dilaton in warped backgrounds

We now consider the perturbation related to the overall size of the compact system, namely the dilaton. It is known that the physical dilaton perturbation that doesn't mix with the graviton(s) can be written in the form [7] (see also [8]):

$$ds^2 = e^{-W(y)^{-2}\gamma(x)} W(y)^2 g_{\mu\nu}(x) dx^\mu dx^\nu + \left(1 + W(y)^{-2}\gamma(x)\right)^2 dy^2 \quad (13)$$

Substituting the above metric in the action (2) (see Appendix for analytic formulas), integrating out total derivatives, throwing out γ -independent parts and keeping terms up to quadratic order, we get:

$$S = \int d^4x dy \sqrt{g} \left\{ -\frac{1}{2} (6M^3 W^{-2}) g^{\mu\nu} \gamma_{,\mu} \gamma_{,\nu} + \mathcal{L}_1 \gamma - \frac{1}{2} \mathcal{L}_2 \gamma^2 \right\} \quad (14)$$

with

$$\mathcal{L}_1 = 2M^3(4W'^2 + 16W''W) + \frac{3}{2}W^2\Phi' \cdot \Phi' + W^2V(\Phi) + 2W^2 \sum_i \lambda_i(\Phi)\delta(y - y_i) \quad (15)$$

$$\mathcal{L}_2 = 2M^3(W^{-2}R_g - 32W'^2W^{-2} + 32W''W^{-1}) + 5\Phi' \cdot \Phi' + 4 \sum_i \lambda_i(\Phi)\delta(y - y_i) \quad (16)$$

At this point, let us work out the integral over the extra dimension of the tadpole term \mathcal{L}_1 of the Lagrangian. This gives:

$$\oint dy \mathcal{L}_1 = -\frac{1}{2} \oint dy W^2 \left[T_\mu^\mu - 2T_5^5 - 16M^3 \left(\frac{W'^2}{W^2} + 4\frac{W''}{W} \right) \right] \quad (17)$$

where we used the energy-momentum tensor components found in the previous section with respect to the unperturbed background metric (5), (6). We can further simplify this quantity if we use the equations (3), (4), (7), (8) which hold for the background metric. The resulting expression is:

$$\oint dy \mathcal{L}_1 = \frac{1}{6} \oint dy W^2 \left[T_\mu^\mu + 4M^3W^{-2}R_g \right] \quad (18)$$

which is exactly zero because of the $a = 2$ constraint (12). This result should have been expected since the perturbation (13) is bound to extremize the effective potential when one evaluates the action using the background equations of motion. However, it is interesting and rather unexpected that the absence of the tadpole term is linked to this particular sum rule of [1]. It is worth mentioning here that in the case that the warp factor is effectively constant ($W \approx 1$), as it was assumed in [6], the condition that the expression (17) vanishes, is identical with the $a = 1$ constraint (11).

Our next task is to read off the mass of the dilaton from the action functional. For this reason we define the canonically normalized dilaton field with mass dimension one $\bar{\gamma}^2 = (6M^3 \oint dy W^{-2}) \gamma^2 \equiv A\gamma^2$. Then the mass of the canonical dilaton $\bar{\gamma}$ is:

$$m^2 = \frac{1}{A} \oint dy \mathcal{L}_2 \quad (19)$$

After a lot of simplifications using the relations (3)-(8) we obtain:

$$m^2 = -\frac{1}{3A} \oint dy (10M^3W^{-2}R_g + \Phi' \cdot \Phi' + 4 \sum_i \lambda_i(\Phi)\delta(y - y_i)) \quad (20)$$

We can further simplify the expression using the $a = 0$ constraint (10) and get a more suggestive result:

$$m^2 = \frac{1}{A} \oint dy (\Phi' \cdot \Phi' - 2M^3 W^{-2} R_g) \quad (21)$$

or equivalently,

$$m^2 = -\frac{1}{A} \left\{ \sum_i \lambda_i(\Phi) + 3M^3 R_g \oint dy W^{-2} \right\} \quad (22)$$

From the second expression it is clear that we cannot have a massive dilaton if the sum of the effective tensions of the branes $\lambda_i(\Phi)$ is exactly zero and at the same time they are kept flat. This is the same conclusion that appeared in [1] regarding the GW mechanism in the RS scenario. Moreover, the absence of tachyonic mass would guarantee the stabilization of the overall size of any system with the above characteristics. By eqs.(21),(22) we get two equivalent conditions:

$$\oint dy (\Phi' \cdot \Phi' - 2M^3 W^{-2} R_g) > 0 \quad (23)$$

$$\sum_i \lambda_i(\Phi) + 3M^3 R_g \oint dy W^{-2} < 0 \quad (24)$$

If one wishes to have flat branes, then the sum of brane tensions should be negative or equivalently one should have a non constant (in y) scalar field configuration. In the case that the above expressions (and thus the mass) vanish, one should look for the higher orders of the effective potential to examine the stability of the system.

4 Conclusions

We have showed that in a general warped metric with identical four-geometries and arbitrary bulk dynamics involving minimally coupled scalar fields, the absence of dilaton tadpoles is related to one particular sum rule of [1]. Moreover, we have calculated the dilaton mass as a function of the sum of the brane tensions and the leftover curvature of the branes. The result agrees with the observation made by [1] that one could not have a massive dilaton for flat four-geometries and zero net brane tensions.

It would be interesting to see what happens with higher than quadratic terms in the dilaton potential and the possible role that the other sum rules of [1] play. Moreover,

one could work out the same calculation for the other moduli in these configurations, the radions, and see if/how the results are modified. Finally, a much more general investigation is needed to obtain the sum rules and their role for the dilaton and radion potentials in models in which the four dimensional geometries are not identical as it happens with cosmological solutions. These are important issues for understanding the dynamics of the dilaton/radions in the extra dimensional models and will be addressed in an other publication [10].

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Appendix

In this appendix we list the Ricci tensor components, the Ricci scalar and the action obtained by the metric:

$$ds^2 = e^{-W(y)^{-2}\gamma(x)} W(y)^2 g_{\mu\nu}(x) dx^\mu dx^\nu + \left(1 + W(y)^{-2}\gamma(x)\right)^2 dy^2 \quad (\text{A.1})$$

The spacetime components of the five dimensional Ricci tensor are:

$$\begin{aligned} R_{\mu\nu} = & R_{g\ \mu\nu} + \frac{g_{\mu\nu}}{2} W^{-2} \square \gamma - \frac{g_{\mu\nu}}{2} \frac{W^{-6} \gamma}{(1 + W^{-2} \gamma)} \gamma_{,\kappa} \gamma^{,\kappa} - \frac{1}{2} \left(\frac{1 - W^{-2} \gamma}{1 + W^{-2} \gamma} \right) W^{-4} \gamma_{,\mu} \gamma_{,\nu} \\ & + \frac{W^{-4} \gamma}{(1 + W^{-2} \gamma)} D_\mu \partial_\nu \gamma - g_{\mu\nu} e^{-W^{-2} \gamma} W'^2 \left(\frac{3 + 4W^{-2} \gamma}{1 + W^{-2} \gamma} \right) \\ & - g_{\mu\nu} e^{-W^{-2} \gamma} \frac{W W''}{(1 + W^{-2} \gamma)} \end{aligned} \quad (\text{A.2})$$

and the (55) component:

$$\begin{aligned} R_{55} = & e^{W^{-2} \gamma} (1 + W^{-2} \gamma) W^{-6} \gamma_{,\mu} \gamma^{,\mu} - e^{W^{-2} \gamma} (1 + W^{-2} \gamma) W^{-4} \square \gamma \\ & - 4(1 + W^{-2} \gamma) W^{-1} W'' - 4(1 + W^{-2} \gamma) W^{-4} W'^2 \end{aligned} \quad (\text{A.3})$$

Finally the Ricci scalar is:

$$\begin{aligned}
R = & e^{W^{-2}\gamma} W^{-2} R_g + e^{W^{-2}\gamma} \left(\frac{1 + 3W^{-2}\gamma}{1 + W^{-2}\gamma} \right) W^{-4} \square \gamma + \frac{1}{2} e^{W^{-2}\gamma} \left(\frac{1 - 3W^{-2}\gamma}{1 + W^{-2}\gamma} \right) W^{-6} \gamma_{,\mu} \gamma^{,\mu} \\
& - 8 \frac{W^{-1} W''}{(1 + W^{-2}\gamma)} - \frac{12W^{-2} W'^2 + 20W^{-4} W'^2 \gamma}{(1 + W^{-2}\gamma)}
\end{aligned} \tag{A.4}$$

In the above expressions the indices are raised and lowered by $g_{\mu\nu}$.

The action (2) then becomes:

$$\begin{aligned}
S = & \int d^4 x dy \sqrt{g} \left\{ 2M^3 \left[e^{-W^{-2}\gamma} W^2 (1 + W^{-2}\gamma) R_g + e^{-W^{-2}\gamma} \left(1 + \frac{5}{2} W^{-2}\gamma + \frac{3}{2} W^{-4}\gamma^2 \right) g^{\mu\nu} D_\mu \partial_\nu \gamma \right. \right. \\
& \left. \left. + e^{-2W^{-2}\gamma} (-8W''W^3 - 12W'^2W^2 - 20W'^2\gamma) \right] \right. \\
& - \frac{1}{2} \frac{W^4}{(1 + W^{-2}\gamma)} e^{-2W^{-2}\gamma} \Phi' \cdot \Phi' - e^{-2W^{-2}\gamma} W^4 (1 + W^{-2}\gamma) V(\Phi) \\
& \left. \left. - e^{-2W^{-2}\gamma} W^4 \sum_i \lambda_i(\Phi) \delta(y - y_i) \right\} \right.
\end{aligned} \tag{A.5}$$

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